

Metric-Palatini gravity unifying local constraints and late-time cosmic acceleration

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We present a novel approach to modified theories of gravity that consists of adding to the Einstein-Hilbert Lagrangian an $f(\mathcal{R})$ term constructed à la Palatini. Using the respective dynamically equivalent scalar-tensor representation, we show that the theory can pass the Solar System observational constraints even if the scalar field is very light. This implies the existence of a long-range scalar field, which is able to modify the cosmological and galactic dynamics, but leaves the Solar System unaffected. We also verify the absence of instabilities in perturbations and provide explicit models which are consistent with local tests and lead to the late-time cosmic acceleration.

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I. INTRODUCTION

In the last ten years modified theories of gravity in which the Einstein-Hilbert Lagrangian is supplemented with additional curvature terms, the so-called $f(R)$ theories [1], have been extensively studied in cosmology. More recently, generalizations of $f(R)$ gravity have been explored, namely, C-theories [2], and nonminimal curvature-matter couplings [3]. There was the hope that the addition of new curvature terms could have an effect on the late-time cosmic dynamics [4], thus providing a gravitational mechanism to explain the accelerated cosmic expansion rate. Though many of the initially proposed models naturally produced the desired late-time acceleration [5], it was soon observed that they were generically affected by serious problems. In particular, in the usual metric variational approach, the modified dynamics of $f(R)$ theories can be interpreted as due to a Brans-Dicke scalar field with parameter $w = 0$ and a non-trivial potential $V(\phi)$. To satisfy the constraints imposed by laboratory and Solar System tests, a perturbative approach indicates that the $w = 0$ scalar should be massive, with an interaction range not exceeding a few millimeters. Such scalars, obviously, cannot have any impact on the cosmology. Metric $f(R)$ theories, therefore, could only have a chance of being viable if by means of non-perturbative effects the scalar somehow managed to hide itself in local experiments while behaving as a long-range field at cosmic scales. Such models are also strongly con-

strained by the observation of cosmological perturbations and none of them seems to perform better than general relativity (GR) with a cosmological constant.

On the other hand, $f(R)$ theories have also been studied in the Palatini approach, where the metric and the connection are regarded as independent fields [6]. In this case, the gravitational dynamics is equivalent to a $w = -3/2$ Brans-Dicke theory with the same potential $V(\phi)$ as in the metric formulation. The $w = -3/2$ theory is characterized by a non-dynamical scalar, which makes it completely different from the other $w \neq -3/2$ theories in which the scalar is dynamical and, therefore, propagates. The non-dynamical nature of the $w = -3/2$ scalar implies that in vacuum the theory turns into general relativity (GR) with an effective cosmological constant Λ_{eff} . This property guarantees the existence of accelerating de Sitter solutions at late times if Λ_{eff} is small. Despite this appealing property, all the Palatini $f(R)$ models studied so far with a small Λ_{eff} lead to microscopic matter instabilities and to unacceptable features in the evolution patterns of cosmological perturbations [6, 12].

In this work we present a new class of modified theories of gravity in which the usual Einstein-Hilbert Lagrangian is supplemented with an $f(\mathcal{R})$ Palatini correction. This type of *hybrid* theory generically arises when perturbative quantization methods are considered on Palatini backgrounds [7] which, on the other hand, have interesting connections with non-perturbative quantum geometries [8].

Already in classical gravitation, one has to specify two connections with physically distinct roles [2], so it is natural to consider that the action depends upon both of the associated curvatures. Metric-Palatini theories admit a non-standard scalar-tensor representation in terms of a dynamical scalar that needs not be massive to pass laboratory and Solar System tests. Microscopic

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matter instabilities are also absent in this model because the field is very weakly coupled to matter. In this theory, therefore, the scalar can play an active role in cosmology without being in conflict with local experiments. We provide explicit examples that illustrate these aspects.

II. SCALAR-TENSOR REPRESENTATION OF METRIC-PALATINI GRAVITY

Consider the action

$$S = \frac{1}{2\kappa^2} \int d^4x \sqrt{-g} [R + f(\mathcal{R})] + S_m, \quad (1)$$

where S_m is the matter action, $\kappa^2 \equiv 8\pi G$, R is the Einstein-Hilbert term, $\mathcal{R} \equiv g^{\mu\nu} \mathcal{R}_{\mu\nu}$ is the Palatini curvature, and $\mathcal{R}_{\mu\nu}$ is defined in terms of an independent connection $\hat{\Gamma}_{\mu\nu}^\alpha$ as $\mathcal{R}_{\mu\nu} \equiv \hat{\Gamma}_{\mu\nu,\alpha}^\alpha - \hat{\Gamma}_{\mu\alpha,\nu}^\alpha + \hat{\Gamma}_{\alpha\lambda}^\alpha \hat{\Gamma}_{\mu\nu}^\lambda - \hat{\Gamma}_{\mu\lambda}^\alpha \hat{\Gamma}_{\alpha\nu}^\lambda$.

The action (1) can be turned into that of a scalar-tensor theory by introducing an auxiliary field A such that

$$S = \frac{1}{2\kappa^2} \int d^4x \sqrt{-g} [R + f(A) + f_A(\mathcal{R} - A)] + S_m, \quad (2)$$

where $f_A \equiv df/dA$. Rearranging the terms and defining $\phi \equiv f_A$, $V(\phi) = Af_A - f(A)$, the action (2) becomes

$$S = \frac{1}{2\kappa^2} \int d^4x \sqrt{-g} [R + \phi\mathcal{R} - V(\phi)] + S_m. \quad (3)$$

Variation of this action with respect to the metric, the scalar ϕ , and the connection leads to

$$R_{\mu\nu} + \phi\mathcal{R}_{\mu\nu} - \frac{1}{2}(R + \phi\mathcal{R} - V)g_{\mu\nu} = \kappa^2 T_{\mu\nu}, \quad (4)$$

$$\mathcal{R} - V_\phi = 0, \quad (5)$$

$$\hat{\nabla}_\alpha(\sqrt{-g}\phi g^{\mu\nu}) = 0, \quad (6)$$

respectively. The solution of Eq. (6) implies that the independent connection is the Levi-Civita connection of a metric $t_{\mu\nu} = \phi g_{\mu\nu}$. This means that $\mathcal{R}_{\mu\nu}$ and $R_{\mu\nu}$ are related by

$$\mathcal{R}_{\mu\nu} = R_{\mu\nu} + \frac{3}{2\phi^2} \partial_\mu \phi \partial_\nu \phi - \frac{1}{\phi} \left(\nabla_\mu \nabla_\nu \phi + \frac{1}{2} g_{\mu\nu} \square \phi \right), \quad (7)$$

which can be used in Eq. (3) to obtain the following scalar-tensor theory

$$S = \int \frac{d^4x \sqrt{-g}}{2\kappa^2} \left[(1 + \phi)R + \frac{3}{2\phi} \partial_\mu \phi \partial^\mu \phi - V(\phi) \right] + S_m. \quad (8)$$

This action differs from the $w = -3/2$ Brans-Dicke theory in the coupling of the scalar to the curvature, which in the $w = -3/2$ theory is of the form ϕR . As we will see, this simple modification will have important physical

consequences. With the expression for $\mathcal{R}_{\mu\nu}$ and Eq. (5), Eq. (4) can be written as

$$(1 + \phi)R_{\mu\nu} = \kappa^2 \left(T_{\mu\nu} - \frac{1}{2} g_{\mu\nu} T \right) + \frac{1}{2} g_{\mu\nu} (V + \square \phi) + \nabla_\mu \nabla_\nu \phi - \frac{3}{2\phi} \partial_\mu \phi \partial_\nu \phi. \quad (9)$$

The scalar field equation can be manipulated in two different ways that illustrate how this theory is related with the $w = 0$ and $w = -3/2$ Brans-Dicke theories. Contracting Eq. (4) with $g^{\mu\nu}$ and using Eq. (5) we find

$$2V - \phi V_\phi = \kappa^2 T + R. \quad (10)$$

Similarly as in the Palatini ($w = -3/2$) case, Eq. (10) tells us that ϕ can be expressed as an algebraic function of the scalar $X \equiv \kappa^2 T + R$, i.e., $\phi = \phi(X)$. In the pure Palatini case, however, ϕ is just a function of T . The right-hand side of Eq. (9), therefore, besides containing new matter terms associated with the trace T and its derivatives, also contains the curvature R and its derivatives. Thus, this theory can be seen as a higher-derivative theory in both the matter fields and the metric. However, an alternative interpretation without higher-order derivatives is possible if R is replaced in Eq. (10) with the relation $R = \mathcal{R} + (3/\phi)\square\phi - (3/2\phi^2)\partial_\mu\phi\partial^\mu\phi$, together with $\mathcal{R} = V_\phi$. One then finds that the scalar field is governed by the second-order evolution equation

$$-\square\phi + \frac{1}{2\phi} \partial_\mu \phi \partial^\mu \phi + \frac{\phi[2V - (1 + \phi)V_\phi]}{3} = \frac{\phi\kappa^2}{3} T. \quad (11)$$

This latter expression shows that, unlike in the Palatini case [6], the scalar field is dynamical and not affected by the microscopic instabilities found in Palatini models with infrared corrections.

III. WEAK-FIELD, SLOW-MOTION BEHAVIOR

The effects of the scalar field ϕ on the Solar System dynamics can be determined by studying the weak-field and slow-motion limit of Eqs. (9) and (11). To do this, we consider an expansion of the metric and the scalar field about a cosmological solution, which sets the asymptotic boundary values, using a quasi-Minkowskian coordinate system, in which $g_{\mu\nu} \approx \eta_{\mu\nu} + h_{\mu\nu}$, with $|h_{\mu\nu}| \ll 1$. Denoting the asymptotic value of ϕ as ϕ_0 and the local perturbation as $\varphi(x)$, to linear order Eq. (11) becomes

$$(\vec{\nabla}^2 - m_\varphi^2)\varphi = \frac{\phi_0\kappa^2}{3}\rho, \quad (12)$$

where $m_\varphi^2 \equiv (2V - V_\phi - \phi(1 + \phi)V_{\phi\phi})/3|_{\phi=\phi_0}$, and we have neglected the time derivatives of φ (slow-motion

regime). Imposing standard gauge conditions, the perturbations $h_{\mu\nu} = g_{\mu\nu} - \eta_{\mu\nu}$ satisfy the following equation

$$-\frac{1}{2}\vec{\nabla}^2 h_{\mu\nu} = \frac{1}{1+\phi_0} \left(T_{\mu\nu} - \frac{1}{2} T \eta_{\mu\nu} \right) + \frac{V_0 + \vec{\nabla}^2 \varphi}{2(1+\phi_0)} \eta_{\mu\nu}, \quad (13)$$

where to this order $T_{00} = \rho$, $T_{ij} = 0$, $T = -\rho$. Far from the sources and assuming spherical symmetry, the solution to Eqs. (12) and (13) lead to ($M = \int d^3x \rho(x)$)

$$\varphi(r) = \frac{2G}{3} \frac{\phi_0 M}{r} e^{-m_\varphi r} \quad (14)$$

$$h_{00}^{(2)}(r) = \frac{2G_{\text{eff}} M}{r} + \frac{V_0}{1+\phi_0} \frac{r^2}{6} \quad (15)$$

$$h_{ij}^{(2)}(r) = \left(\frac{2\gamma G_{\text{eff}} M}{r} - \frac{V_0}{1+\phi_0} \frac{r^2}{6} \right) \delta_{ij}, \quad (16)$$

where we have defined the effective Newton constant G_{eff} and the post-Newtonian parameter γ as

$$G_{\text{eff}} \equiv \frac{G}{1+\phi_0} [1 - (\phi_0/3) e^{-m_\varphi r}], \quad (17)$$

$$\gamma \equiv \frac{1 + (\phi_0/3) e^{-m_\varphi r}}{1 - (\phi_0/3) e^{-m_\varphi r}}. \quad (18)$$

As is clear from the above expressions, the coupling of the scalar field to the local system depends on the amplitude of the background value ϕ_0 . If ϕ_0 is small, then $G_{\text{eff}} \approx G$ and $\gamma \approx 1$ regardless of the value of the effective mass m_φ^2 . When this mass squared becomes negative, the exponential terms in the above expressions become cosinus. This contrasts with the result obtained in the metric version of $f(R)$ theories [9]. In that case one finds $\varphi = (2G/3)(M/r)e^{-m_f r}$, $G_{\text{eff}} \equiv G(1 + e^{-m_f r}/3)/\phi_0$, and $\gamma \equiv (1 - e^{-m_f r}/3)/(1 + e^{-m_f r}/3)$, which requires a large mass $m_f^2 \equiv (\phi V_{\phi\phi} - V_\phi)/3$ to make the Yukawa-type corrections negligible in local experiments. This may be achieved in specific models implementing the chameleon mechanism [10].

We note that the massless limit of the results derived in this section are in complete agreement with the analysis of [11] for general massless scalar-tensor theories.

IV. LATE-TIME COSMIC SPEEDUP

As a specific example of modified cosmological dynamics, we consider the spatially flat Friedman-Robertson-Walker (FRW) metric

$$ds^2 = -dt^2 + a^2(t) d\mathbf{x}^2, \quad (19)$$

where $a(t)$ is the scale factor. The Ricci scalar is given by $R = 6(2H^2 + \dot{H})$, where $H = \dot{a}(t)/a(t)$ is the Hubble parameter, and $\dot{a} \equiv da/dt$. With this metric, Eq. (9) yields the following evolution equations

$$3H^2 = \frac{1}{1+\phi} \left[\kappa^2 \rho + \frac{V}{2} - 3\dot{\phi} \left(H + \frac{\dot{\phi}}{4\phi} \right) \right], \quad (20)$$

$$2\dot{H} = \frac{1}{1+\phi} \left[-\kappa^2(\rho + P) + H\dot{\phi} + \frac{3}{2} \frac{\dot{\phi}^2}{\phi} - \ddot{\phi} \right] \quad (21)$$

The scalar field equation (11) becomes

$$\ddot{\phi} + 3H\dot{\phi} - \frac{\dot{\phi}^2}{2\phi} + \frac{\phi}{3} [2V - (1+\phi)V_\phi] = -\frac{\phi\kappa^2}{3}(\rho - 3P). \quad (22)$$

The qualitative behavior of the scalar field can be read directly from Eq. (22) by writing the latter as follows

$$\ddot{\phi} + 3H\dot{\phi} - \frac{\dot{\phi}^2}{2\phi} + M_\phi^2(T)\phi = 0, \quad (23)$$

where $T = -(\rho - 3P)$ and we have defined $M_\phi^2(T)$ as

$$M_\phi^2(T) \equiv m_\phi^2 - \frac{1}{3}\kappa^2 T = \frac{1}{3}[2V - (1+\phi)V_\phi - \kappa^2 T], \quad (24)$$

which despite the notation needs not be a positive function. Aside from the $\dot{\phi}^2/\phi$ term, which is only important when ϕ is very rapidly changing, Eq. (23) represents a massive scalar field on an FRW background. During the matter dominated era ($T = -\rho$), the cosmic fluid contributes to the oscillation frequency of the scalar, while the friction term $3H\dot{\phi}$ forces a progressive damping of its amplitude. At late times, when $T \approx 0$, the sign of m_ϕ^2 determines whether the field oscillates or grows exponentially fast. This aspect is model dependent and will be considered next.

A. Two models

We now propose two models that are consistent at Solar System and cosmological scales, and which are constructed on grounds of mathematical simplicity. A quantitative analysis of the high-precision astrophysical and cosmological data will be carried out elsewhere to find and constrain more general families of models within the metric-Palatini framework.

The first model arises by demanding that matter and curvature satisfy the same relation as in GR. Taking

$$V(\phi) = V_0 + V_1 \phi^2, \quad (25)$$

Eq. (10) automatically implies $R = -\kappa^2 T + 2V_0$. As $T \rightarrow 0$ with the cosmic expansion, this model naturally evolves into a de Sitter phase, which requires $V_0 \sim \Lambda$ for consistency with observations. If V_1 is positive, the de Sitter regime represents the minimum of the potential. The effective mass for local experiments, $m_\varphi^2 = 2(V_0 - 2V_1\phi)/3$, is then positive and small as long as $\phi < V_0/V_1$. For sufficiently large V_1 one can make the field amplitude small enough to be in agreement with Solar System tests. It is interesting that the exact de Sitter solution is compatible with dynamics of the scalar field in this model.

A second model can be found by rewriting Eq. (20) as

$$\left(H + \frac{\dot{\phi}}{2(1+\phi)}\right)^2 = \frac{\kappa^2 \rho + V/2}{3(1+\phi)} - \frac{\dot{\phi}^2}{4\phi(1+\phi)^2} \quad (26)$$

and looking for late-time solutions, with

$$H + \frac{\dot{\phi}}{2(1+\phi)} = \tilde{H}_0 = \text{constant}, \quad (27)$$

which leads to

$$a\sqrt{1+\phi} = a_0 e^{\tilde{H}_0 t}. \quad (28)$$

When $\rho \rightarrow 0$, the evolution equations and $\dot{\tilde{H}}_0 = 0$ lead to

$$\left(\tilde{H}_0^2 - \frac{V}{6}\right)^2 = 9\tilde{H}_0^2 \phi \left[\frac{V}{6(1+\phi)} - \tilde{H}_0^2\right], \quad (29)$$

from which one obtains

$$V(\phi) = \frac{3\tilde{H}_0^2}{(1+\phi)} \left[2 + 11\phi \pm 3\phi\sqrt{5-4\phi}\right]. \quad (30)$$

Remarkably, there are no free parameters in this model except for its amplitude \tilde{H}_0^2 , which should be of the same order as the currently estimated cosmological constant, and the sign in front of the squared root. For the minus sign, we find that m_ϕ^2 is positive and of order $\sim \tilde{H}_0^2$ for small ϕ , which provides another model with a long-range scalar consistent with local tests and late-time cosmic speedup.

B. Cosmological perturbations

It should be noted that due to the appearance of an effective sound speed for matter perturbations, most pure Palatini- $f(R)$ models can be ruled out as dark energy candidates [12]. In the metric-Palatini approach presented here, however, such Laplacian instabilities are absent. Let us describe the relative perturbation of the matter distribution in a dust-dominated universe by $\delta \equiv \delta\rho/\bar{\rho}$, where $\bar{\rho}$ is the smooth background and $\delta\rho$ the inhomogeneous part. Allowing also the metric and the scalar field to fluctuate, one can consistently consider the evolution of perturbations. At subhorizon scales, that is described by

$$\ddot{\delta} + 2H\dot{\delta} = 4\pi G_{\text{eff}}\bar{\rho}\delta, \quad G_{\text{eff}} \equiv \frac{1-\phi/3}{1+\phi}G. \quad (31)$$

In accordance with (17), the effective Newton's constant becomes now time-dependent at cosmological scales. This modifies the growth rate of perturbations, and opens the possibility to distinguish between models with similarly accelerating background, e.g. [13]. In addition, effective anisotropic stresses will appear, which could be

detected in future weak lensing surveys such as the Euclid mission.

One may also verify that in vacuum the fluctuations $\delta\phi$ in the effective field ϕ at small scales propagate with the speed of light. Thus, despite the nonstandard coupling of the scalar-field, its perturbations behave physically.

V. CONCLUSIONS

In this work we have considered a class of modified gravity actions where the corrections to the Einstein-Hilbert term are given by a function of the Ricci scalar constructed from an *a priori* metric-independent connection. We have shown that the theory admits a scalar-tensor representation which possesses a new mechanism to pass the Solar System constraints even if the scalar field is very light. This can be seen from the first order Post-Newtonian corrections to the metric given in Eqs. (14)-(16). If the current cosmic amplitude ϕ_0 is sufficiently small, $|\phi_0| \ll 1$, a long-range scalar field able to affect the cosmic and galactic dynamics can also be compatible with the Solar System dynamics.

Motivated by mathematical simplicity, we have presented two cosmological models with asymptotically de Sitter behavior. In both cases the early time evolution also seems consistent with the well-known radiation and matter dominated phases of the cosmic evolution. The potential (30) had only one adjustable parameter, which is set by the observed scale of acceleration. The simpler potential (25) included also the parameter fixing the magnitude of the quadratic correction. A quantitative analysis of structure formation and CMB anisotropies can thus be used to test the viability of this and other models.

Before concluding, we point out that a modified gravitational potential of the form $\Phi = -GM[1 + \alpha_0 \exp(-r/r_0)] / (1 + \alpha_0)r$, with $\alpha_0 = -0.9$ and $r_0 \approx 30$ kpc [14] provides a very good description of the flat rotational curves of a significant sample of galaxies. The form of the weak-field potentials of the metric-Palatini hybrid model considered in this work share an interesting formal resemblance with this proposal. Thus, in addition to passing the Solar System constraints, the theory considered in this work may open new possibilities to approach, in the same theoretical framework, of both dark matter and dark energy problems. Also such aspects as future singularities [15] would be interesting study in the hybrid theory. Further work along these lines is presently underway.

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